

The budget set

Lecture notes

What this lecture is about. Every choice a consumer makes is limited by what they can afford. Before we ask what people *want* (preferences, next week), we describe precisely what they *can* have. This set of options is the budget set. We describe it with a single straight line, learn three ways to write that line, and then see how it moves when income, prices, taxes or pricing plans change.

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1 Rational consumer choice

Economists assume that consumers choose *consistently*: faced with the same options in different contexts, they make the same kind of choice. This assumption is the basis of the **theory of rational consumer choice**. Individual choices then add up to **market demand**, the demand curve we used in the last lectures.

Prices are given: a single consumer cannot change them. Choosing then happens in two steps:

1. Describe everything the consumer can afford: the **budget set** (this lecture).
2. Rank those options by what the consumer likes: **preferences** (next lecture).

Bundle and flows

A **bundle** (x, y) lists an amount of each good: x units of the first good and y units of the second. Quantities and budgets are **flows**: they are measured per period of time (per week, per month). A quantity without its time unit has no meaning, and a monthly budget must be compared with monthly quantities.

2 The budget line

Take two goods, x and y , with prices p_x and p_y , and a budget B per period. Spending all of the budget means

$$p_x x + p_y y = B.$$

Budget line and budget set

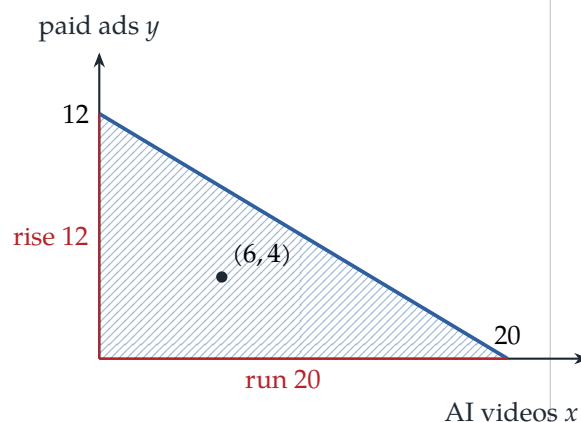
The **budget line** is the set of all bundles that cost exactly B . The **budget set** (or affordable set) is the set of all bundles that cost at most B : the budget line together with every bundle below it.

Example: an AI influencer account

You run an AI influencer account. Each month you spend a budget of $B = €60$ on two things: AI videos (x) at $p_x = €3$ each, and paid ads (y) at $p_y = €5$ each.

The ends of the line. If you spend everything on videos you get $B/p_x = 60/3 = 20$ videos; if you spend everything on ads you get $B/p_y = 60/5 = 12$ ads. The budget line joins $(20, 0)$ and $(0, 12)$.

Is $(6, 4)$ affordable? It costs $3 \cdot 6 + 5 \cdot 4 = 38 \leq 60$, so yes: it lies inside the budget set, and €22 are left over.



The slope: rise over run. Going along the line from the y -end to the x -end, the line falls by the whole y -intercept (the rise) over the whole x -intercept (the run):

$$\frac{\text{rise}}{\text{run}} = \frac{B/p_y}{B/p_x} = \frac{p_x}{p_y}, \quad \text{so the slope is } -\frac{p_x}{p_y}.$$

In the example the rise is 12 and the run is 20, so the slope is $-12/20 = -3/5 = -p_x/p_y$.

The slope is an opportunity cost

The size of the slope, p_x/p_y , tells you how many units of y you must give up to get one more unit of x . One more AI video (€3) costs you $3/5$ of a paid ad (€5). The budget does not matter for the slope: only the ratio of the two prices does.

3 Three ways to write the budget line

The same line can be written in three equivalent forms. Each is useful for something different.

form	general	example ($p_x = 3, p_y = 5, B = 60$)
standard form	$p_x x + p_y y = B$	$3x + 5y = 60$
slope–intercept form	$y = \frac{B}{p_y} - \frac{p_x}{p_y} x$	$y = 12 - \frac{3}{5}x$
intercept form	$\frac{x}{B/p_x} + \frac{y}{B/p_y} = 1$	$\frac{x}{20} + \frac{y}{12} = 1$

The standard form shows the spending. The slope–intercept form ($y = \dots$) shows the y -intercept and the slope at a glance and is the one we draw from. The intercept form shows both ends of the line directly: the budget divided by each price.

4 Shifts and rotations

The budget line moves in two basic ways.

Shift and rotation

Shift: if the budget changes and prices stay the same, the line moves parallel to itself (to the right if B rises, to the left if it falls). The slope $-p_x/p_y$ does not change.

Rotation: if one price changes, the line turns around the end that belongs to the other good. If p_x falls, the x -end moves out and the y -end stays where it is.

Example: three budget lines

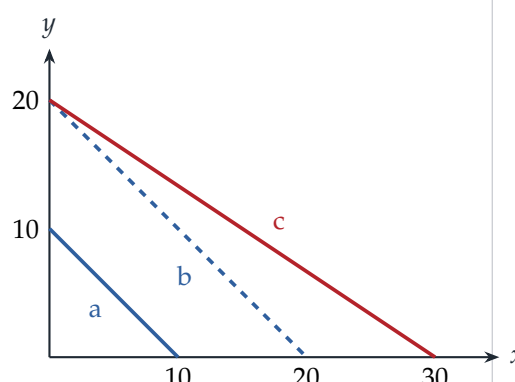
Still with AI videos (x) and paid ads (y):

	budget, prices	ends	line
a	$B = 60, p_x = p_y = 6$	10 and 10	$y = 10 - x$
b	$B = 120, p_x = p_y = 6$	20 and 20	$y = 20 - x$
c	$B = 120, p_x = 4, p_y = 6$	30 and 20	$y = 20 - \frac{2}{3}x$

a → b: income doubles, prices stay: a **shift**. Both lines have slope -1 .

b → c: videos get cheaper: a **rotation** around $(0, 20)$. The slope becomes $-20/30 = -2/3 = -4/6$: an AI video now costs two thirds of an ad.

Any budget gives the same slope for line c: what matters is only $p_x/p_y = 2/3$. Indeed $4x + 6y = 120$ and $2x + 3y = 60$ are the same line.



5 The composite good

Most decisions are about one good against everything else. We can capture this with two goods if we let y stand for “everything else”, measured in euro. This idea goes back to Alfred Marshall.

Composite good and numéraire

The **composite good** y is the money spent on all other goods. One unit of it is €1, so its price is $p_y = 1$. A good whose price is set to 1 and in which all other prices are counted is called the **numéraire**.

With $p_y = 1$ the budget line becomes very simple: $y = B - p_x x$. The y -intercept is the budget itself and the slope is $-p_x$.

Example: the main model for the rest of the lecture

You run an AI influencer account with $B = €60$ a month. AI videos cost $p_x = €3$; everything else is the composite good ($p_y = 1$). The budget line is

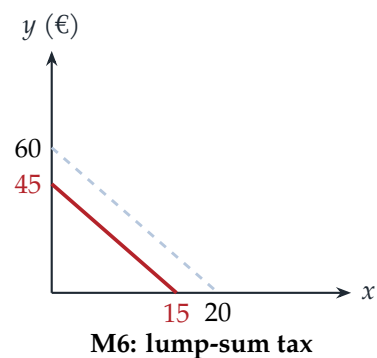
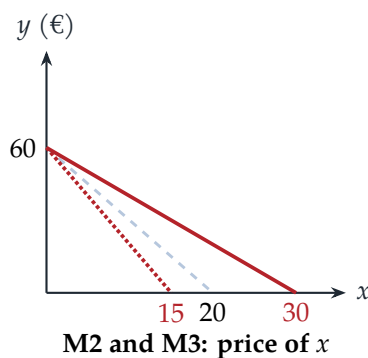
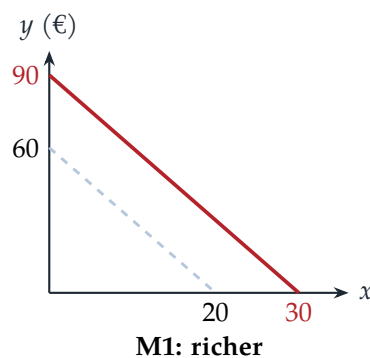
$$y = 60 - 3x \quad \text{or} \quad \frac{x}{20} + \frac{y}{60} = 1,$$

with ends 20 (videos) and 60 (euro of everything else), and slope -3 : each video costs €3 of everything else.

6 One story, all the moves

We now change one thing at a time in the main model $y = 60 - 3x$. In every figure the dashed line is the main model and the red line is the new budget line.

	change	new line	ends	movement
M1	budget rises to $B = €90$	$y = 90 - 3x$	30 and 90	shift right
M2	a video costs $p_x = €2$	$y = 60 - 2x$	30 and 60	rotation out around $(0, 60)$
M3	a video costs $p_x = €4$	$y = 60 - 4x$	15 and 60	rotation in around $(0, 60)$
M4	$B = 120, p_x = 6, p_y = 2$	$y = 60 - 3x$	20 and 60	no change
M5	a tax of €1 per video	$y = 60 - 4x$	15 and 60	rotation in (as M3)
M6	a lump-sum tax of €15	$y = 45 - 3x$	15 and 45	shift left



Why does M4 change nothing? Multiply every price and the budget by the same number and the line is unchanged: $6x + 2y = 120$ is the same as $y = 60 - 3x$. Only *relative* prices and the budget measured in those prices (real income) matter.

Two taxes, two movements. A per-unit tax raises the price of every video, so it rotates the line (M5 works exactly like M3). A lump-sum tax takes a fixed amount from the budget whatever you buy, so it shifts the line in parallel (M6).

Check yourself

In M2, which bundle is affordable both before and after the price change, at the y -axis? Why does a per-unit tax and a lump-sum tax of the same size leave you with different options?

7 Kinked budget lines and price plans

If the price per unit changes at some quantity, the budget line is made of two straight pieces with a corner, a **kink**. We find each piece separately.

Example: a surcharge after 10 videos (M7)

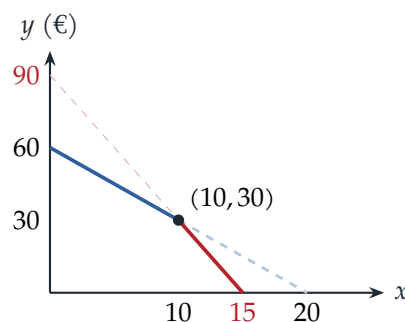
The first 10 videos cost €3 each; every video after that costs €6.

Up to 10 videos the old line holds: $y = 60 - 3x$. At $x = 10$ you have $y = 30$ left. This is the kink $(10, 30)$.

After the kink the slope is -6 , so the line is $y = a - 6x$ for some intercept a . It must pass through the kink:

$$30 = a - 6 \cdot 10 \Rightarrow a = 90, \quad y = 90 - 6x.$$

It reaches the x -axis at $90/6 = 15$ videos (without the surcharge it would be 20).



Example: pay-as-you-go or a Pro plan (M8)

Instead of paying €3 per video, you can take a Pro plan: €12 a month plus €1.50 per video.

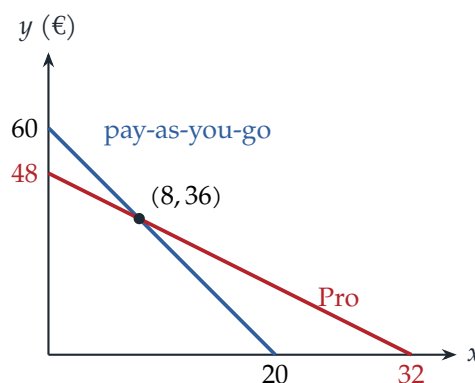
Pay-as-you-go: $y = 60 - 3x$, at most 20 videos.

Pro plan: the fee leaves $60 - 12 = 48$, so $y = 48 - 1.5x$, at most $48/1.5 = 32$ videos.

Where do they cross?

$$60 - 3x = 48 - 1.5x \Rightarrow x = 8, y = 36.$$

With fewer than 8 videos a month pay-as-you-go leaves you more for everything else; above 8 the Pro plan does. Your budget set is everything below either line.



8 Endowments

Sometimes a consumer starts with goods rather than money. What they own is their **endowment**. Its market value is their budget, and they can always simply keep what they have.

Endowment budget line

With an endowment of goods, the budget is the market value of the endowment. The budget line always passes through the endowment point. When the price of the good you own changes, the line rotates around the endowment point.

Example: headphones from brands (N1, N2)

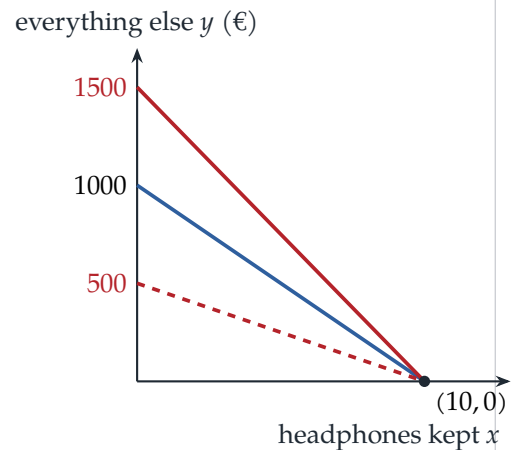
Brands send your influencer account 10 headphones a month, and you can resell them at $p_x = €100$ each. You have no cash. Let x be the headphones you keep and y everything else (composite good, $p_y = 1$).

Budget: $B = 10 \cdot 100 = €1000$, so $y = 1000 - 100x$. Keeping all ten is the endowment point $(10, 0)$; selling all ten gives €1000.

Resale price €150: $B = 1500$ and $y = 1500 - 150x$.

Resale price €50: $B = 500$ and $y = 500 - 50x$.

In both cases the line rotates around $(10, 0)$: whatever the price, you can always keep your ten headphones.



9 Summary

- The budget set contains every bundle that costs at most B ; the budget line, $p_x x + p_y y = B$, contains the bundles that cost exactly B .
- The ends of the line are the budget over each price, B/p_x and B/p_y . The slope, rise over run, is $-p_x/p_y$: the opportunity cost of one more unit of x in units of y .
- A change in the budget shifts the line; a change in one price rotates it. Scaling all prices and the budget together changes nothing.
- With the composite good ($p_y = 1$) the line is $y = B - p_x x$.
- A per-unit tax rotates the line; a lump-sum tax shifts it.
- When the price per unit changes at some quantity, the line has a kink: find each piece separately, using the kink point.
- With an endowment, the budget is the value of what you own, and price changes rotate the line around the endowment point.

Further reading. Frank, *Microeconomics and Behavior*, chapter 3, pp. 55–63 (week 3 page, additional readings; the password is given in class).